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A multidimensional inverse radiation problem of estimating the strength of a heat source in participating media

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Abstract

We consider an inverse radiation problem of determining the time-varying strength of a heat source, which mimics flames in a furnace, from temperature measurements in three-dimensional participating media where radiation and conduction occur simultaneously. The inverse radiation problem is posed as a minimization problem of the least-squares criterion, which is solved by a conjugate gradient method employing the adjoint equation to determine the descent direction. The discrete ordinate S_4 method (M.F. Modest, Radiative Heat Transfer, McGraw-Hill, New York, 1993) is employed to solve the radiative transfer equation and its adjoint equation accurately. The performance of the present technique of inverse analysis is evaluated by several numerical experiments, and it is found to solve the inverse radiation problem accurately without a priori information about the unknown function to be estimated.

1. Introduction

In the present investigation, we consider a method of determining the time-varying strength of a heat source in radiatively participating media from the temperature measurements in the domain. The heat source in the present case may be thought of as a model of flames in a furnace where the major mechanism of heat transfer is radiation. The radiative heat transfer in furnaces is a very complicated phenomenon as tiny suspending particles scatter, absorb and emit radiation. The governing equation for this process is given as an integro-differential equation in a phase space, called the radiative transfer equation [1,2]. Since the radiation affects the temperature field, the actual mode of heat transfer in most cases is combined radiation and conduction or convection. In the present work, we consider heat transfer by combined conduction with radiation through participating media capable of absorbing, emitting, and scattering thermal radiation. In this case, it is necessary to solve an energy conservation equation that explicitly provides the local temperature which determines the blackbody intensity in the radiative transfer equation. On the other hand, the divergence of the radiative flux that is present as a source term in the energy conservation equation is obtained only after solving the radiative transfer equation. Thus, the problems are always implicit in temperature, and therefore require iterative procedure which makes the modeling of these problems challenging. Regarding the problem under consideration in the present investigation, if the strength of the heat source is known, one solves the energy conservation equation with the heat source term and the radiative transfer equation simultaneously to obtain the temperature field in the domain. This is the direct problem. Conversely, the strength of the heat source can be determined with the help of extra conditions such as temperature measurements at certain interior points of the domain. Such a problem is one of the inverse problems and can be regarded as discovering the cause from a known result. These inverse problems are ill-posed in the sense that small perturbations in the observed functions may result in large changes in the corresponding solutions [3,4]. The ill-posed nature requires special numerical techniques having regularization

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Nomenclature

C_p	heat capacity
$d^n(t)$	conjugate direction
F	performance function (Eq. (7))
$G(t)$	heat source function
I	radiation intensity
I_b	blackbody intensity ($\equiv \sigma_b T^4/\pi$)
J	adjoint radiation intensity
k	thermal conductivity (W/m K)
MO	number of measurement points
$\hat{\boldsymbol{n}}$	unit inward normal vector
P	adjoint temperature
$\boldsymbol{q}_r$	radiative heat flux
$\hat{\boldsymbol{s}}$	unit vector into a given direction
t	time
t_f	final time
T	temperature
T^*	observed temperature

Greek symbols

$\delta(x)$	Dirac delta function
$\delta_n(x)$	function defined in Eq. (2)
δF	variation of the performance function F
δG	variation of the heat source function
δI	variation of the radiation intensity
δT	variation of the temperature field
ϵ	emissivity
κ	absorption coefficient
ρ	optimal step length (Eq. (25))
σ	scattering coefficient
σ_b	Stefan–Boltzmann constant, $\sigma_b = 5.670 \times 10^{-8}$ W/m^2 K^4
φ^n	parameter defined in Eq. (18)
$\boldsymbol{\Omega}$	solid angle
∇F	gradient of the performance function

Superscripts

$*$	measured variable
$\dagger$	location of heat source

Subscripts

m	measurement point
mCG	modified conjugate gradient

properties to stabilize the results of calculations. Recently the conjugate gradient methods have been employed in the solution of inverse heat conduction problems [5,6], and found to be very efficient.

Contrary to the inverse heat conduction problems, the inverse radiation problems have not been addressed frequently [7]. We may mention two classes of inverse radiation problems. One class is the determination of the radiative parameters from various types of measurements [8,9]. In Park and Yoon [9], they developed a conjugate gradient method of solving three-dimensional inverse radiation problems which allows one to estimate the radiative parameters from the measurement of temperature. Since the number of the unknown parameters is finite in this case, the gradient of the performance function was conveniently obtained by employing the direct differentiation method. The other class of the inverse radiation problem is the determination of unknown functions, e.g. the time-varying strength of a heat source in the present investigation, from measurements in the domain. Since the function determination is an infinite dimensional problem in parameter space, the appropriate method of obtaining the gradient of the performance function is to adopt the adjoint equation [5]. In the present investigation, the inverse radiation problem of estimating the time-varying strength of a heat source is posed as a minimization problem of the least-squares criterion, which is solved by a conjugate gradient method employing the adjoint equation to determine the descent direction. The radiative transfer equation and its adjoint are solved by the discrete ordinate S_4 method [1].

2. The system

We consider a rectangular furnace of size (1 m × 1 m × 1 m) (Fig. 1(a)) containing a participating medium with opaque and diffusively reflecting boundaries. Heat transfer in this system is contributed by conduction as well as radiation with absorption, scattering and emission. The governing equation for the temperature field is as follows:

$$\rho C_p \frac{\partial T}{\partial t} = k\nabla^2 T - \nabla \cdot \boldsymbol{q}_r + G(t)\delta_n(x - x^\dagger)\delta_n(y - y^\dagger) \times \delta_n(z - z^\dagger), \tag{1}$$

where ρ (= 0.4 kg/m^3) is the density of the medium, C_p (= 1100 J/kg K) is the heat capacity and k (= 44 W/m K) is the effective thermal conductivity. In Eq. (1), $G(t)$ denotes the strength of heat source and the function $\delta_n(x - x^\dagger)$, which approximates the heat source at $x = x^\dagger$, is defined by:

$$\delta_n(x - x^\dagger) = \frac{n}{2\cosh^2(n(x - x^\dagger))} \tag{2}$$

and becomes the Dirac delta function as n approaches infinity. In the present work, we take $n = 20$ with the heat source location $(x^\dagger, y^\dagger, z^\dagger) = (0.35, 0.35, 0.425)$. The relevant boundary conditions for Eq. (1) are:

- at all boundaries; $T = 800$ K. (3)

The divergence of radiative heat flux $\nabla \cdot \boldsymbol{q}_r$ in Eq. (1) is determined by the following equation:

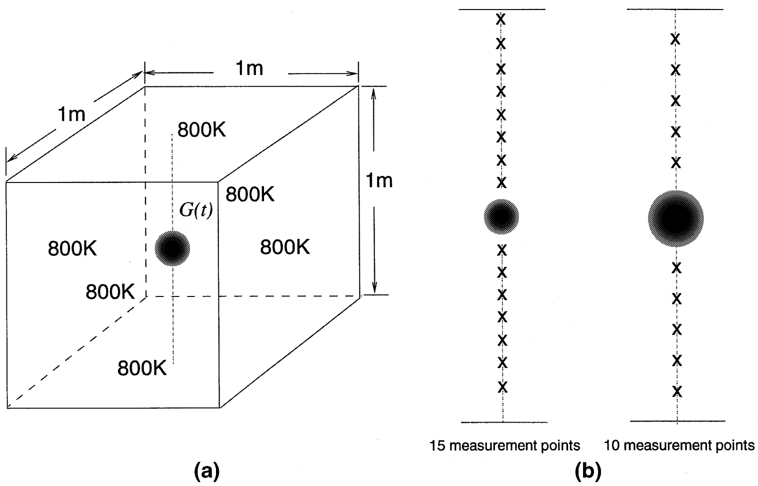

Fig. 1. (a) The system with a heat source. (b) Typical temperature measurement points located along the center axis of the furnace (15 measurement points and 10 measurement points, respectively).

$$\nabla \cdot \boldsymbol{q}_r = 4\pi\kappa\left(\frac{\sigma_b}{\pi}T^4 - \frac{1}{4\pi}\int_{4\pi} I \, d\boldsymbol{\Omega}\right), \quad (4)$$

where κ is the absorption coefficient, σ_b is the Stefan-Boltzmann constant, $\boldsymbol{\Omega}$ is the solid angle and I is the radiation intensity. The governing equation of I for participating media with absorption, emission and isotropic scattering is:

$$\nabla \cdot (\hat{\boldsymbol{s}}I) + (\kappa+\sigma)I - \kappa I_b - \frac{\sigma}{4\pi}\int_{4\pi} I(\hat{\boldsymbol{r}}, \hat{\boldsymbol{s}}') \, d\boldsymbol{\Omega}' = 0, \quad (5)$$

where $\hat{\boldsymbol{s}}$ is a unit vector in the beam direction, σ the scattering coefficient, I_b is the Planck function (black-body intensity). The relevant boundary conditions are:

- at the wall; $I(\boldsymbol{r}, \hat{\boldsymbol{s}}) = \epsilon I_b + \frac{1-\epsilon}{\pi}\int_{\hat{\boldsymbol{n}}\cdot\hat{\boldsymbol{s}}'<0} |\hat{\boldsymbol{n}}\cdot\hat{\boldsymbol{s}}| \times I(\boldsymbol{r}\cdot\hat{\boldsymbol{s}}') \, d\boldsymbol{\Omega}' \quad (\hat{\boldsymbol{n}}\cdot\hat{\boldsymbol{s}} > 0), \quad (6)$

where $\hat{\boldsymbol{n}}$ is the inward normal vector into the cavity, $\hat{\boldsymbol{s}}'$ the unit vector in the incoming beam direction and ϵ is the wall emissivity. Eqs. (1) and (5) are coupled through the terms $\nabla \cdot \boldsymbol{q}_r$ and I_b, and must be solved iteratively to yield the radiation and temperature fields. The computational procedure is as follows:

1. Assume the temperature field.
2. Calculate I_b using the given temperature field.
3. Solve the radiative transfer equation (Eq. (5)) using the S_4 method [1] to obtain the radiation intensity I.
4. The divergence of the radiative heat flux is determined by Eq. (4).
5. Solve Eq. (1) using a finite volume method to obtain the temperature field.
6. If the radiation and the temperature fields are not converged, go to the step 2. Otherwise, move to the next time step.

3. Solution of the inverse radiation problem using a conjugate gradient method

The temperature field in the furnace varies according to the heat source function $G(t)$. Therefore, $G(t)$ can be estimated by using the measured values of temperature field at certain locations. The performance function for the identification of $G(t)$ is expressed by the sum of square residuals between the calculated and observed temperature as follows:

$$F = \frac{1}{2}\sum_{m=1}^{MO}\int_0^{t_f} [T(x_m, y_m, z_m, t) - T^*(x_m, y_m, z_m, t)]^2 \, dt, \quad (7)$$

where $T(x_m, y_m, z_m, t)$ is the calculated temperature, $T^*(x_m, y_m, z_m, t)$ the measured temperature at the same location at the same time (x_m, y_m, z_m, t), and MO is the total number of measurement points. To minimize the performance function (7) using a conjugate gradient method, we need the gradient of F, ∇F, defined by

$$\delta F(G) \equiv F(G+\delta G) - F(G) = \langle \nabla F, \delta G \rangle = \int_0^{t_f} \nabla F \delta G \, dt, \quad (8)$$

where t_f, the final time, is 1.0 s. If we take larger values of t_f, it becomes easier to estimate $G(t)$, but the computer time required for the estimation increases. The function ∇F can be obtained by introducing the adjoint variables $P(\boldsymbol{x},t)$ and $J(\boldsymbol{x},\boldsymbol{\Omega})$ such that

$$F = \frac{1}{2}\int_0^{t_f}\sum_{m=1}^{MO}[T(x_m,y_m,z_m,t) - T^*(x_m,y_m,z_m,t)]^2 \, dt - \int_0^{t_f}\int_{\boldsymbol{x}} P\left[\rho C_p \frac{\partial T}{\partial t} - k\nabla^2 T + \nabla\cdot \boldsymbol{q}_r - G(t)\delta_n(x-x^\dagger)\delta_n(y-y^\dagger)\delta_n(z-z^\dagger)\right] d\boldsymbol{x}\, dt - \int_0^{t_f}\int_{\boldsymbol{x}}\int_{\boldsymbol{\Omega}} J\left[\nabla\cdot(\hat{\boldsymbol{s}}I) + (\kappa+\sigma)I - \kappa I_b - \frac{\sigma}{4\pi}\int_{4\pi} I(\boldsymbol{r},\hat{\boldsymbol{s}}')\, d\boldsymbol{\Omega}'\right] d\boldsymbol{\Omega}\, d\boldsymbol{x}\, dt. \tag{9}$$

The variation of F, δF, is given by the following equation:

$$\delta F = \int_0^{t_f}\sum_{m=1}^{MO}[T(x_m,y_m,z_m,t) - T^*(x_m,y_m,z_m,t)] \times \delta T(x_m,y_m,z_m,t)\, dt - \int_0^{t_f}\int_{\boldsymbol{x}} P\left[\rho C_p\frac{\partial \delta T}{\partial t} - k\nabla^2\delta T + 4\pi\kappa\left(\frac{\sigma_b}{\pi}4T^3\delta T - \frac{1}{4\pi}\int_{4\pi}\delta I\, d\boldsymbol{\Omega}\right) - \delta G(t)\delta_n(x-x^\dagger)\delta_n(y-y^\dagger)\delta_n(z-z^\dagger)\right] d\boldsymbol{x}\, dt - \int_0^{t_f}\int_{\boldsymbol{x}}\int_{\boldsymbol{\Omega}} J\left[\nabla\cdot(\hat{\boldsymbol{s}}\delta I) + (\kappa+\sigma)\delta I - \kappa\frac{\sigma_b}{\pi}4T^3\delta T - \frac{\sigma}{4\pi}\int_{4\pi}\delta I\, d\boldsymbol{\Omega}'\right] d\boldsymbol{\Omega}\, d\boldsymbol{x}\, dt. \tag{10}$$

Integrating δF by parts both in space and time, and exploiting the boundary conditions for T and I, the gradient of F, ∇F, defined in Eq. (8) is found to be

$$\nabla F = \int_{\boldsymbol{x}} P(\boldsymbol{x},t)\delta_n(x-x^\dagger)\delta_n(y-y^\dagger)\delta_n(z-z^\dagger)\, d\boldsymbol{x} \tag{11}$$

while the governing equations for the adjoint variables $P(\boldsymbol{x},t)$ and $J(\boldsymbol{x},\boldsymbol{\Omega})$ are as follows

$$\rho C_p\frac{\partial P}{\partial t} + k\nabla^2 P - P(4\pi\kappa)\left(\frac{\sigma_b}{\pi}\right)(4T^3) + \kappa\frac{\sigma_b}{\pi}(4T^3) \times \int_{\boldsymbol{\Omega}} J\, d\boldsymbol{\Omega} + \sum_{m=1}^{MO}[T(x_m,y_m,z_m,t) - T^*(x_m,y_m,z_m,t)] \times \delta(x-x^\dagger)\delta(y-y^\dagger)\delta(z-z^\dagger) = 0, \tag{12}$$

- $t = t_f, \quad P(\boldsymbol{x},t) = 0,$ (13)
- at all boundaries; $P(\boldsymbol{x},t) = 0,$ (14)

$$\nabla\cdot(\hat{\boldsymbol{s}}J) - (\kappa+\sigma)J + \frac{\sigma}{4\pi}\int_{4\pi} J\, d\boldsymbol{\Omega} + \kappa P = 0, \tag{15}$$

- at all boundaries; $J(\boldsymbol{x},\boldsymbol{\Omega}) = 0,$ (16)

where $\delta(x)$ in Eq. (12) is the Dirac delta function. Eq. (12) is solved by using a finite volume method, and the discrete ordinate S_4 method for the angular discretization and the diamond-scheme for the spatial discretization are adopted to solve Eq. (15). Once the gradient function ∇F is obtained by using Eq. (11), the strength of the heat source $G(t)$ can be estimated by employing the following conjugate gradient procedure [5].

Step 1. Assume $G(t)$.

Step 2. Define the scalar φ:

$$\varphi^{(i)} = 0 \quad \text{if } i = 0, \tag{17}$$

$$\varphi^{(i)} = \frac{\int_0^{t_f}[\nabla F(G^{(i)})]^2\, dt}{\int_0^{t_f}[\nabla F(G^{(i-1)})]^2\, dt} \quad (i \geqslant 1). \tag{18}$$

Step 3. Define the conjugate direction $d^{(i)}$:

$$d^{(0)} = \nabla F(G^{(0)}), \tag{19}$$

$$d^{(i)} = \nabla F(G^{(i)}) + \varphi^{(i)}d^{(i-1)} \quad \text{if } i \geqslant 1. \tag{20}$$

Step 4. Determine the optimal step length $\rho^{(i)}$ such that

$$\frac{\partial}{\partial\rho}F(G^{(i)} - \rho d^{(i)}) = 0 \quad \text{for } \rho = \rho^{(i)}. \tag{21}$$

Step 5. Set

$$G^{(i+1)}(t) = G^{(i)}(t) - \rho^{(i)}d^{(i)}(t). \tag{22}$$

Step 6. If

$$\int_0^{t_f}[G^{(i+1)}(t) - G^{(i)}(t)]^2\, dt < \epsilon, \text{ stop} \tag{23}$$

Otherwise, set $i = i+1$, go to Step 2.

The optimal step length $\rho^{(i)}$ in the Step 4 is obtained by assuming quadratic variation of F with respect to ρ. Denoting the directional derivative of T at $G(t)$ in the direction of $d(t)$ by δT, we have

$$T(x_m,y_m,z_m,t;G^{(i)} - \rho d^{(i)}) = T(x_m,y_m,z_m,t;G^{(i)}) - \rho\delta T(x_m,y_m,z_m,t). \tag{24}$$

Substituting Eq. (24) into Eq. (7), partially differentiating it with respect to ρ and setting the resulting equation equal to zero, the value of ρ that minimizes $F(G^{(i)} - \rho d^{(i)})$ is obtained as

$$\rho = \int_0^{t_f}\sum_{m=1}^{MO}[T(x_m,y_m,z_m,t) - T^*(x_m,y_m,z_m,t)]\delta T(x_m,y_m,z_m,t)\, dt \Bigg/ \int_0^{t_f}\sum_{m=1}^{MO}[\delta T(x_m,y_m,z_m,t)]^2\, dt. \tag{25}$$

The sensitivity equation which determines $\delta T(x,y,z,t)$ is given by the following set of equations

$$\rho C_p \frac{\partial \delta T}{\partial t} - k\nabla^2 \delta T + 4\pi\kappa\left(\frac{\sigma_b}{\pi}4T^3\delta T - \frac{1}{4\pi}\int_{4\pi}\delta I \ \mathrm{d}\boldsymbol{\Omega}\right) - d^{(i)}(t)\delta_n(x-x^\dagger)\delta_n(y-y^\dagger)\delta_n(z-z^\dagger) = 0, \quad (26)$$

- $t = 0, \quad \delta T(x,y,z,t) = 0,$ (27)
- at all boundaries; $\delta T(x,y,z,t) = 0,$ (28)

$$\nabla\cdot(\hat{\boldsymbol{s}}\delta I) + (\kappa+\sigma)\delta I - \kappa\left(\frac{\sigma_b}{\pi}4T^3\delta T\right) - \frac{\sigma}{4\pi}\int_{4\pi}\delta I \ \mathrm{d}\boldsymbol{\Omega} = 0, \quad (29)$$

- at all boundaries; $\delta I(\boldsymbol{r},\hat{\boldsymbol{s}}) = \frac{1-\epsilon}{\pi}\int_{\hat{\boldsymbol{n}}\cdot\hat{\boldsymbol{s}}'<0} |\ \hat{\boldsymbol{n}}\cdot\hat{\boldsymbol{s}}\ | \times \delta I(\boldsymbol{r}\cdot\hat{\boldsymbol{s}}') \ \mathrm{d}\boldsymbol{\Omega}' \quad (\hat{\boldsymbol{n}}\cdot\hat{\boldsymbol{s}} > 0).$ (30)

Eqs. (26) and (29) are solved by using the finite volume method [4] and the S_4 method, respectively.

4. Modified conjugate gradient method [10]

The conjugate gradient method described in the previous section yields accurate profiles of the heat strength $G(t)$ after sufficient number of iterations, except its value at the final time $G(t_f)$. According to the starting condition for the adjoint variable P, Eq. (13), we find that the gradient function ∇F is zero at the final time (cf. Eq. (11)). Therefore, the conjugate direction $d(t)$ is also zero at $t = t_f$ (cf. Eqs. (19) and (20)), and according to Eq. (22) the heat source function at the final time $G(t_f)$ remains at its initial guess $G^0(t_f)$. The difficulty encountered at the final time t_f can be alleviated by employing the following modification suggested by Alifanov [10]. We seek a continuously differentiable function $G(t)$ such that

$$G(t) = \int^t \frac{\mathrm{d}G(t')}{\mathrm{d}t'} \ \mathrm{d}t'. \quad (31)$$

From Eqs. (8) and (11), the variation of the performance function δF may be rewritten as:

$$\delta F = \int_0^{t_f}\int_{\boldsymbol{x}} P(\boldsymbol{x},t)\delta G(t)\delta_n(x-x^\dagger)\delta_n(y-y^\dagger) \times \delta_n(z-z^\dagger) \ \mathrm{d}\boldsymbol{x} \ \mathrm{d}t. \quad (32)$$

Integrating Eq. (32) by parts with respect to t,

$$\delta F = -\int_0^{t_f}\frac{\mathrm{d}\delta G(t)}{\mathrm{d}t}\int_{t_f}^{t}\int_{\boldsymbol{x}} P(\boldsymbol{x},t')\delta_n(x-x^\dagger)\delta_n(y-y^\dagger) \times \delta_n(z-z^\dagger) \ \mathrm{d}\boldsymbol{x} \ \mathrm{d}t' \ \mathrm{d}t. \quad (33)$$

Therefore, the derivative of F with respect to $\mathrm{d}G/\mathrm{d}t$ is given by the following expression

$$\nabla F\left(\frac{\mathrm{d}G}{\mathrm{d}t}\right) = -\int_{t_f}^{t}\int_{\boldsymbol{x}} P(\boldsymbol{x},t')\delta_n(x-x^\dagger)\delta_n(y-y^\dagger) \times \delta_n(z-z^\dagger) \ \mathrm{d}\boldsymbol{x} \ \mathrm{d}t' \ \mathrm{d}t. \quad (34)$$

Then, we take the conjugate direction as follows:

$$d^{(i)}(t) = \int_0^t D^{(i)}(t') \ \mathrm{d}t', \quad (35)$$

where

$$D^{(i)} = \nabla F\left(\frac{\mathrm{d}G}{\mathrm{d}t}\right)^{(i)} + \varphi^{(i)}D^{(i-1)}. \quad (36)$$

Since $d^{(i)}(t_f)$, given by Eq. (35), is nonzero, the modified conjugate gradient method yields an accurate prediction of $G(t_f)$ contrary to the previous regular conjugate gradient method. On the other hand, from Eq. (35) it can be seen that $d^{(i)}(0) = 0$. Then, for the same reason with the regular conjugate gradient method, the modified conjugate gradient method will not improve the initial value of the heat source function $G(0)$. In the present investigation, this dilemma is overcome by combining the regular and modified conjugate gradient method sequentially. At the first stage, we employ the modified conjugate gradient method for a certain number of iterations until a reasonably good estimation of the end value $G(t_f)$ is attained. Afterwards, the regular conjugate gradient method is adopted using the estimation of the modified conjugate gradient method as the initial approximation until a converged profile is obtained.

5. Results

The present method, which solves the inverse radiation problem of estimating the time-varying strength of a heat source in a furnace from temperature measurements in the domain, has been tested using several sets of simulated measurements $T^\dagger(x_m, y_m, z_m, t)$, and the estimated strength of the heat source is compared with the exact one. We consider two different cases of heat source function $G(t)$, as depicted in Figs. 2(a) and (b). The equations of $G(t)$ for the two cases shown in Fig. 2 are as follows:

(a) $G(t) = 1600 \ (0 \leqslant t \leqslant 0.044),$
$G(t) = 180{,}000t - 6320 \ (0.044 \leqslant t \leqslant 0.11),$
$G(t) = -180{,}000t + 33{,}280 \ (0.11 \leqslant t \leqslant 0.176),$
$G(t) = 1600 \ (0.176 \leqslant t \leqslant 0.22),$ (37)

(b) $G(t) = 1600 \ (0 \leqslant t \leqslant 0.055),$
$G(t) = 20{,}000 \ (0.055 \leqslant t \leqslant 0.165),$
$G(t) = 1600 \ (0.165 \leqslant t \leqslant 0.22).$ (38)

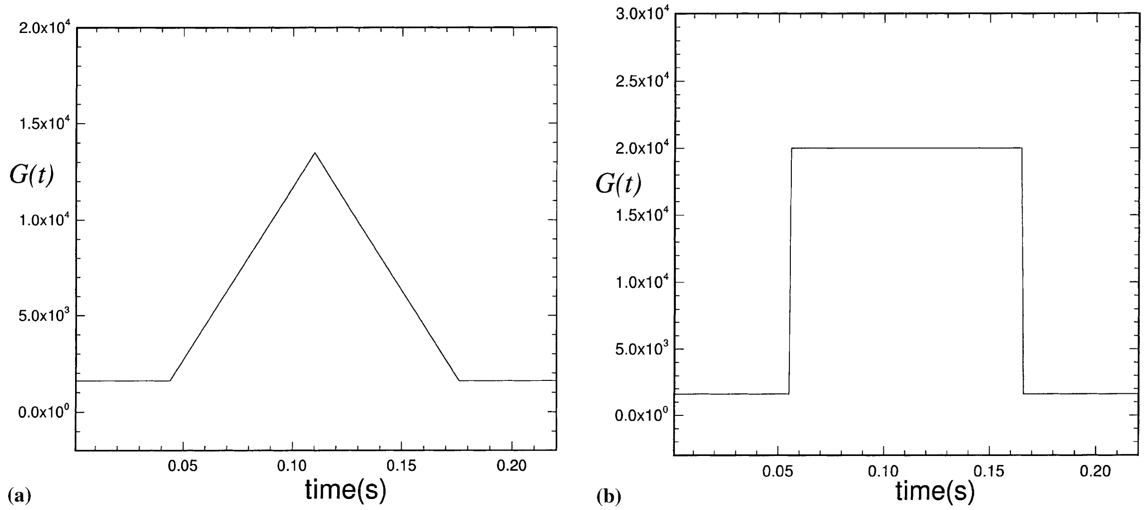

Fig. 2. Shapes of heat source function $G(t)$ considered in the present investigation.

For all tests presented in this section, the initial approximation of $G(t)$ in the conjugate gradient method is taken to be 5000. (constant). The following definition of estimation error is adopted to compare the quality of the estimation.

$$\text{Error} = \frac{\|G_{\text{estimated}} - G_{\text{exact}}\|_{L_2}^2}{\|G_{\text{exact}}\|_{L_2}^2}, \tag{39}$$

where $\|\cdot\|_{L_2}$ is the usual L_2-norm. The simulated measurements containing measurement errors are generated by adding Gaussian distributed random errors to the computed exact temperatures as follows

$$T_{\text{measured}}(= T^*) = T_{\text{exact}} + \omega\sigma, \tag{40}$$

where σ determines the noise level and ω is a random number between $-2.576 \leqslant \omega \leqslant 2.576$. In fact, σ is the standard deviation of measurement errors which are assumed to be the same for all measurements, and ω is the Gaussian distributed random error. The above range of the ω value corresponds to the 99% confidence bound for the temperature measurement. We adjust σ such that the relative measurement error is zero, 3% and 5%, respectively.

As explained in Section 4, the regular conjugate gradient method does not improve the final value $G(t_f)$ while the modified conjugate gradient method has the same difficulty with the initial value $G(0)$. The combined iteration scheme [6] is employed to overcome this dilemma. At the first stage, we employ the modified conjugate gradient method for a certain number of iterations until a reasonably good estimation of the final value $G(t_f)$ is attained. Afterwards, the regular conjugate gradient method is adopted using the estimation of the modified conjugate gradient method as the initial approximation to get the final converged profile. The error of $G(t_f)$ with the modified conjugate gradient method is defined by

$$E_{\text{mCG}} = \sum_{i=1}^{3} \frac{|\, G^{(n)}(t_f) - G^{(n-i)}(t_f)\,|}{|\, G^{(n)}(t_f)\,|} \tag{41}$$

and the iteration of the modified conjugated gradient is stopped when $E_{\text{mCG}} < 0.01$.

As the first test, we consider an idealized situation in which there are no measurement errors ($\sigma = 0$). The temperature measurements are assumed to be done continuously by using 15 thermocouples located as shown in Fig. 1(b). Figs. 3(a) and (b) show the estimated profiles of the heat source function $G(t)$ for the two cases of Fig. 2. The combined iteration scheme is employed to obtain good estimations of both the initial and final values of $G(t)$. The iteration number of the modified conjugate gradient method and that of the regular conjugate gradient method are indicated as well as the estimation error. The estimated profiles are in good agreement with the exact heat source function over the whole domain, with the estimation error being 5.328×10^{-2} for the case (a) and 4.034×10^{-2} for the case (b). Fig. 4 shows the effect of the number of measurement points on the accuracy of the estimated heat source function. Instead of the 15 measurement points depicted in Fig. 1(b), the 10 measurement points are employed to obtain temperature recordings. Comparing the result of Fig. 4 with that of Fig. 3(a), we find that the reduction of the number of measurement points deteriorates the accuracy of the estimation, especially near the initial time.

Finally, the effect of measurement errors on the accuracy of the estimation is investigated. When there are measurement errors, the following discrepancy principle is adopted as the stopping criterion for the iterative

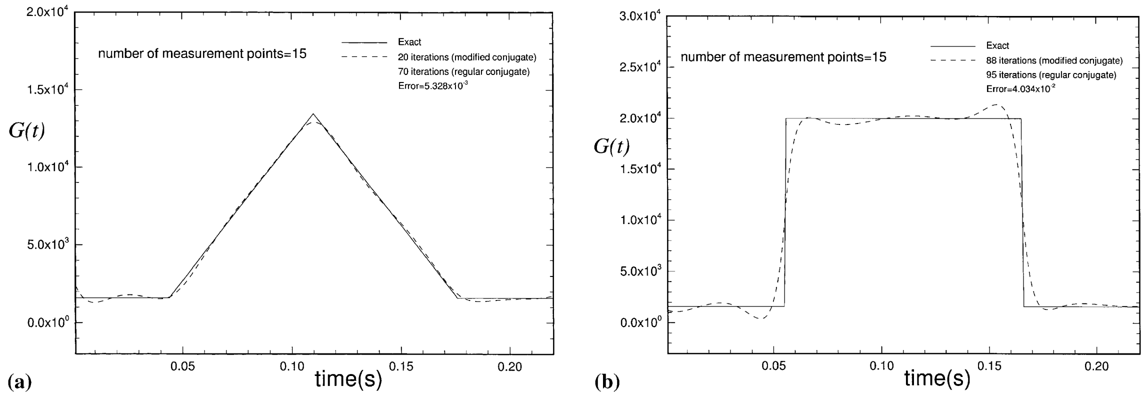

Fig. 3. The estimated profiles of the heat source function $G(t)$ when the combined iteration scheme is employed. (a) Case (a) of Fig. 2. (b) Case (b) of Fig. 2.

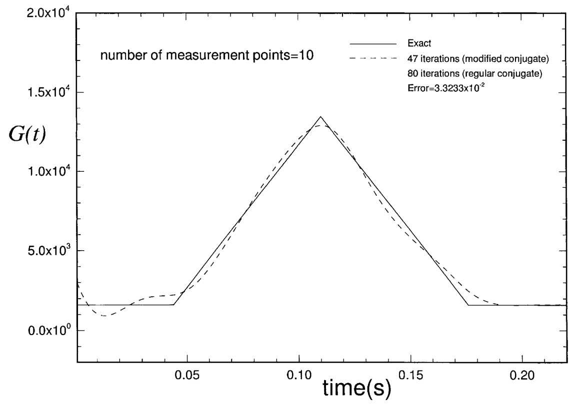

Fig. 4. The estimated profile of $G(t)$ when the number of measurement points is reduced to 10.

procedure of the conjugate gradient method [6]. Assuming the measurement errors to be the same for all thermocouples, i.e.

$$T(x_m, y_m, z_m, t) - T^*(x_m, y_m, z_m, t) \approx \sigma. \tag{42}$$

Introducing this result into Eq. (7), we find

$$F \approx \frac{1}{2}\int_0^{t_f} \sum_{m=1}^{MO} \sigma^2 \, dt \equiv \epsilon^2. \tag{43}$$

Then the discrepancy principle for the stopping criterion is taken as

$$F < \epsilon^2. \tag{44}$$

If the function F has a minimum value that is larger than ϵ^2, the following criterion is used to stop the iteration

$$F(G^{(i+1)}) - F(G^{(i)}) < \epsilon_1, \tag{45}$$

where ϵ_1 is a prescribed small number. Figs. 5(a) and (b) show the estimated heat source function $G(t)$ when the relative measurement error is 3% (Fig. 5(a)) and 5% (Fig. 5(b)), respectively. As expected, the accuracy of estimation deteriorates as the measurement error

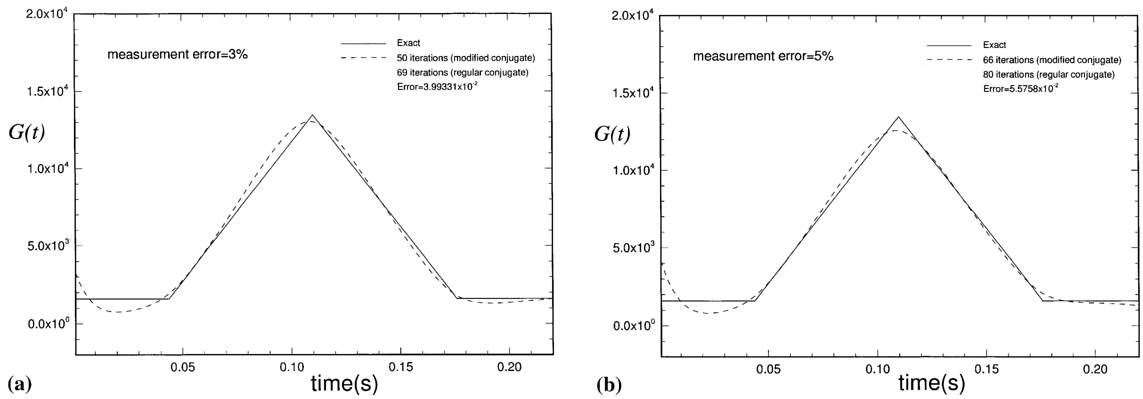

Fig. 5. The effect of measurement errors on the accuracy of estimation. (a) Relative measurement error = 3%. (b) Relative measurement error = 5%.

increases. It is also shown that the estimated profiles of $G(t)$ near the initial time are sensitive to measurement errors. The reverse iteration procedure, where the regular conjugate gradient iteration is performed before the modified conjugate gradient iteration, is tried to remedy the error near the initial time, but it does not make any difference.

6. Conclusion

The inverse radiation problem of estimating the unknown strength of a time-varying heat source from the temperature measurement within participating media is investigated by employing the conjugate gradient method. The gradient of the performance function is obtained by using the adjoint equations. The radiative transfer equation and its adjoint equation are solved by means of the S_4 method. The performance of the present technique of inverse radiation problem is evaluated by several numerical experiments, and it is found to solve the inverse radiation problem accurately without a priori information about the unknown function to be estimated.

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